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TEMPERATURE FIELDS AND STRESSES IN BODIES WITH DISCONTINUOUS PARAMETERS

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UDC 539.3

Bodies with discontinuous parameters are utilized extensively as structural elements in different areas of modern engineering. Among them are thermally sensitive, piecewise-homogeneous and multistage bodies; bodies with piecewise-constant coefficients of heat elimination from their surfaces, with cutouts, holes, gaps, of finite dimensions. The physico-mechanical characteristics of multistage structural elements can be described as a single whole for the whole body by using asymmetric unit functions, while bodies with a continuous inhomogeneity and bodies with temperature-dependent properties (thermally sensitive bodies) are approximated by using these functions. The desired functions (the temperature field t, the stress tensor components σ_{ij}, and the displacement vector u_i) for bodies with cutouts, gaps, holes, of finite dimensions can be continued into the domain enclosing the cutout, gap, hole; in an infinite domain in one dimension, respectively. For the bodies under consideration this permits writing integrodifferential (generalized problem) or differential equations (classical problem) of heat conduction and thermoelasticity with discontinuous and singular coefficients. For bodies with discontinuous coefficients of heat elimination, the boundary conditions can be written with discontinuous coefficients. Investigations executed in this scientific direction are generalized in the monographs [1-4] and analyzed in [5]. Here we examine the analysis of further investigations performed in the area of the heat conduction and thermoelasticity of bodies with discontinuous parameters by using the apparatus of generalized functions.

Let us consider an anisotropic inhomogeneous body occupying a domain V and having the temperature t_0 in the undeformed and unstressed state. A system of differential equations is obtained in [6] for the generalized interconnected dynamic problem of the thermoelasticity of an anisotropic inhomogeneous body under the assumption that the relaxation time of the heat flux does not change as a function of the coordinates, which is valid for metals [7]. If the relaxation time $\tau_r(M)$ depends on the coordinates, i.e., the generalized heat-conduction law has the form

$$lq_i = -\lambda_{ij}^t (M) t_{,j} \quad (M \in V,\ i,\ j = 1,\ 2,\ 3), \tag{1}$$

the heat-conduction equation for an anisotropic inhomogeneous body will then be written for $\dot{t}/_{\tau=0} = 0$ as

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$$\int_0^\tau [\Omega(M,\ \tau,\ \zeta)\,\lambda_{ij}^t(M)(t(M,\ \zeta))_{,j}]_{,i}\,d\zeta = t_0\beta_{ij}(M)\,\dot{e}_{ij} + c_v(M)\,\dot{t} - w_t, \qquad (2)$$

where

$$\beta_{ij}(M) = c_{ijkl}(M)\,\alpha_{kl}^t(M),$$

$$\Omega(M,\ \tau,\ \zeta) = \frac{\exp\left(\dfrac{\zeta-\tau}{\tau_r(M)}\right)}{\tau_r(M)}, \quad l = 1 + \tau_r(M)\frac{\partial}{\partial\tau}.$$

The equations of motion in displacements have the form [7]

$$[c_{ijkl}(M)\,u_{k,l}]_{,j} = \rho(M)\,\ddot{u}_i + [\beta_{ij}(M)\,\theta]_{,j}, \qquad (3)$$

where $\theta = t - t_0$; $\tau_r(M) = \tau_r(x_1,\ x_2,\ x_3)$.

The integrodifferential equation (2) and the differential equations (3) form a complete system of equations for a generalized interconnected dynamic problem of thermoelasticity of an anisotropic inhomogeneous body. Needed in addition to the equations presented for the formulation of the thermoelasticity problem are the boundary and initial conditions. For the classical thermoelasticity problem they are presented in [4]. For the generalized problem the boundary condition for heat transfer of the third kind is written in such a manner

$$n_i\int_0^\tau \Omega(P,\ \tau,\ \zeta)\,\lambda_{ij}^t(P)(t(P,\ \zeta))_{,j}\,d\zeta + \alpha_s[t(P,\ \tau) - t_c^s(P,\ \tau)] = 0, \quad P\in S. \qquad (4)$$

The boundary conditions of the second and third kind result from (4) for $\alpha_S t_C^S = q$, $\alpha_S = 0$; $\alpha_S \to \infty$, respectively.

The equations of the generalized unconnected dynamical problem of thermoelasticity in cylindrical and spherical coordinates have the form

$$\int_0^\tau \Big\{\frac{\partial}{\partial r}\left[\Omega_\lambda(M,\ \tau,\ \zeta)\frac{\partial t}{\partial r}\right] + \frac{\Omega_\lambda(M,\ \tau,\ \zeta)}{r}\frac{\partial}{\partial r} +$$

$$+ r^{-2}\frac{\partial}{\partial\varphi}\left[\Omega_\lambda(M,\ \tau,\ \zeta)\frac{\partial t}{\partial\varphi}\right] +$$

$$+ \frac{\partial}{\partial z}\left[\Omega_\lambda(M,\ \tau,\ \zeta)\frac{\partial t}{\partial z}\right]\Big\}\,d\zeta = c_v(M)\,\dot{t} - w_t, \qquad (5)$$

$$A_r\frac{\partial}{\partial r}[2\mu(M)\,e_{rr} + \lambda(M)\,e - \beta(M)\,\theta] + 2A_\varphi\frac{\partial}{\partial\varphi}[\mu(M)\,e_{r\varphi}] +$$

$$+ 2A_z\frac{\partial}{\partial z}[\mu(M)\,e_{rz}] = \rho(M)\,\ddot{u} - H_r \quad (r\varphi z,\ uvw),$$

where

$$\beta(M) = \alpha_t(M)[3\lambda(M) + 2\mu(M)],$$

$$\Omega_\lambda(M,\ \tau,\ \zeta) = \frac{\lambda_t(M)}{\tau_r(M)}\exp\left(\frac{\zeta-\tau}{\tau_r(M)}\right),$$

$$A_r = A_z = 1, \quad A_\varphi = \frac{1}{r}, \quad H_r = \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r}, \quad H_\varphi = \frac{2}{r}\sigma_{r\varphi}, \quad H_z = \frac{\sigma_{rz}}{r};$$

$$\int_0^\tau \Big\langle \frac{\partial}{\partial r}\left[\Omega_\lambda(M,\ \tau,\ \zeta)\frac{\partial t}{\partial r}\right] + \frac{2}{r}\Omega_\lambda(M,\ \tau,\ \zeta)\frac{\partial t}{\partial r} +$$

$$+ r^{-2}\Big\{\frac{\partial}{\partial\varphi}\left[\Omega_\lambda(M,\ \tau,\ \zeta)\frac{\partial t}{\partial\varphi}\right] + \operatorname{ctg}\varphi\,\Omega_\lambda(M,\ \tau,\ \zeta)\frac{\partial t}{\partial\varphi} +$$

$$+ \sin^{-2}\varphi\frac{\partial}{\partial\psi}\left[\Omega_\lambda(M,\ \tau,\ \zeta)\frac{\partial t}{\partial\psi}\right]\Big\}\Big\rangle\,d\zeta = c_v(M)\,\dot{t} - w_t, \qquad (6)$$

$$A_r \frac{\partial}{\partial r}[2\mu(M)e_{rr}+\lambda(M)e-\beta(M)\theta]+2A_\varphi \frac{\partial}{\partial\varphi}[\mu(M)e_{r\varphi}]+$$

$$+2A_\psi \frac{\partial}{\partial\psi}[\mu(M)e_{r\psi}]=\rho(M)\ddot{u}-H_r\ (r\varphi\psi,\ uvw),$$

where

$$A=1,\ \ A_\varphi=\frac{1}{r},\ \ A_\psi=\frac{1}{r\sin\varphi},$$

$$H_r=\frac{1}{r}(2\sigma_{rr}-\sigma_{\varphi\varphi}-\sigma_{\psi\psi}+\sigma_{r\varphi}\operatorname{ctg}\varphi),$$

$$H_\varphi=\frac{1}{r}[3\sigma_{r\varphi}+(\sigma_{\varphi\varphi}-\sigma_{\psi\psi})\operatorname{ctg}\varphi],$$

$$H_\psi=\frac{1}{r}(3\sigma_{r\psi}+2\sigma_{\psi\varphi}\operatorname{ctg}\varphi),\quad \mu(M)=\frac{E(M)}{2[(1+\nu(M)]},$$

and the remaining notation is given in [4].

Continuous quantities are in the square brackets in the equations presented here. For bodies with a continuous inhomogeneity they should be differentiated as continuous functions. For a piecewise-homogeneous body, i.e., a body consisting of separate parts with different but constant physicomechanical characteristics within the limits of each of them, one of the factors in the products in the square brackets of the equations presented above is a piecewise-constant function while the other is piecewise-continuous. These functions have common points of discontinuity of the first kind. Differentiation of the product of such functions is realized according to the rule

$$(\Phi\Psi)'=\Phi\Psi'+\Phi'\Psi\mp[\Phi][\Psi]\delta_\pm(z-z_1), \tag{7}$$

where $[\Phi]$, $[\Psi]$ are jumps in the functions Φ and Ψ at the common point of discontinuity of the first kind $z = z_1$, $\delta_\pm(\zeta) = dS_\pm(\zeta)/d\zeta$.

If the function $f(z)$ is such that the unilateral limits $f(z_1 \pm 0)$, $f'(z_1 \pm 0)$, ..., $f^{(n)}(z_1 \pm 0)$ $(b < z_1 < d)$ exist, then the following relationships hold [4]:

$$f(z)\delta_\pm(z-z_1)=f(z_1\pm 0)\delta_\pm(z-z_1), \tag{8}$$

$$f(z)\delta'_\pm(z-z_1)=f(z_1+0)\delta'_\pm(z-z_1)-f'(z_1\pm 0)\delta_\pm(z-z_1). \tag{9}$$

Setting $f(z) = S_\pm(z - z_1)$ in the relationships (8) and (9), we obtain

$$S_\pm(z-z_1)\delta_\pm(z-z_1)=\begin{cases}\delta_+(z-z_1),\\ 0,\end{cases}$$
$$S_\pm(z-z_1)\delta'_\pm(z-z_1)=\begin{cases}\delta'_+(z-z_1),\\ 0.\end{cases} \tag{10}$$

If the physicomechanical characteristics of a multilayered body as a single whole are represented in the form

$$p(z)=p_1+\sum_{k=1}^{m-1}(p_{k+1}-p_k)S_\pm(z-z_k),$$

then any of the combinations is also represented in the same form. Here p_k are the physicomechanical characteristics of the k-th element of the packet, which are constant within the limits of the domain occupied by each element separately; m is the quantity of elements in the packet, and z_k is the coordinate of the plane conjugate to the k-th and (k + 1)-th element of the packet.

Then taking account of the rule for differentiating (7) and (10), we arrive at the following heat conduction equation with discontinuous and singular coefficients for the classical problem of a multilayered isotropic body, say:

$$\Delta t=\frac{\dot{t}}{a_1}-\frac{w_t}{(\lambda_t)_1}+\varepsilon_1\dot{e}+\sum_{k=1}^{m-1}\Big\{[(a_{k+1}^{-1}-a_k^{-1})\,\dot{t}+ \\ +(\varepsilon_{k+1}-\varepsilon_k)\,\dot{e}-((\lambda_t)_{k+1}^{-1}-(\lambda_t)_k^{-1})\,w_t]\,S_-(z-z_k)+ \\ +\left(1-\frac{(\lambda_t)_{k+1}}{(\lambda_t)_k}\right)\frac{\partial t}{\partial z}\Big|_{z=z_k+0}\delta_-(z-z_k)\Big\}, \tag{11}$$

where

$$\varepsilon_k=t_0\frac{\beta_k}{(\lambda_t)_k};\quad \beta_k=(\alpha_t)_k(3\lambda_k+2\mu_k);\quad a_k=\frac{(\lambda_t)_k}{(c_v)_k}.$$

For $\varepsilon_i = 0$ ($i = 1, 2, \ldots, m$) Eq. (11) is used in [8, 9] to study the temperature fields in piecewise-homogeneous isotropic bodies heated by heat sources or the external medium. A heat-conduction equation with discontinuous and singular coefficients is written in [10] for a piecewise-homogeneous orthotropic body, a method is proposed to determine the temperature fields in adjoined diverse orthotropic rectangular wedges for a discontinuous boundary condition of the first kind.

The purpose of [11] is to give a theoretical foundation to the scheme utilized earlier [4] for obtaining heat conduction and thermoelasticity equations of piecewise-homogeneous bodies with discontinuous and singular coefficients.

The temperature stresses in a cylindrical shell and plate-strip with piecewise-constant heat-elimination coefficients from the side surfaces are studied in [12, 13]. The papers [14, 18] are devoted to an investigation of the quasistationary temperature fields and the temperature stresses they cause in homogeneous and piecewise-homogeneous semiinfinite domains for boundary conditions of the third kind with discontinuous coefficients of heat elimination from the boundary surface.

Investigation of the temperature fields and stresses in multistage thin-walled structure elements is of important practical value. In particular, such problems occur in the study of technological welding processes of plates and shells of different thickness and rods of different diameters; the thermal strength of metal–glass junctures of the stems of electrovacuum instrument shells containing metal cylindrical step current leads; in the investigation and analysis of measurement errors by low-temperature resistance thermometers because of the heat influx along the current leads and the protective armature. Differential equations with coefficients of the impulsive heat conduction function are derived in [2, 4, 19-21] for isotropic thin plates, cylindrical shells, and rods with heat elimination and heat sources taken into account, and the quasistatic problem of thermoelasticity for circular and rectangular plates. A nonlinear heat-conduction problem is considered in [22] for multistage thin-walled structure elements heated by radiation. Differential equations with singular coefficients are obtained to determine the integral characteristics of the Kirchhoff variable. On the basis of the equations deduced, single closed solutions are obtained for the whole domain of definition for nonstationary and stationary problems of heat conduction, quasistatic and static thermoelasticity problems of multistage circular plates, and two-stage semiinfinite platelets.

The study of the temperature stresses in multistage thin-walled elements on the basis of the spatial heat conduction and thermoelasticity problem equations is of considerable interest.

The method of continuation of functions is effective in the solution of heat conduction and thermoelasticity problems for such bodies as well as bodies with cutouts of finite size. It is used in [23-29] to determine the temperature fields and stresses in plates with square, rectangular, or a system of rectangular cutouts, and the temperature stresses in a half-strip.

A thin foreign layer is formed during the treatment of metal details by concentrated energy fluxes [30, 31]. The model of a body bonded by thin layers of thickness 2d [4] can be used in determining the temperature stresses in such structure elements. Let us introduce the reduced heat conduction $\Lambda_0 = 2(\lambda_t)_0 d$, the density $R_0 = 2\rho_0 d$, and the shear stiffness $g_0 = 2G_0 d$. Since the layer thickness is considerably less than their intervening distance, we perform a passage to the limit as $d \to 0$ in (2) and (3) for an isotropic body and retain λ_0, R_0, g_0 constant. We consequently obtain the following equations of the generalized problem:

$$\int_0^{\tau}\left[\Omega_1 t_{,i}+\Lambda_0\sum_{k=1}^{m}(M_0-K_\lambda M_1)(t_{,i})^*\delta(x_3-z_k)\right]_{,i}d\zeta=-w_t+$$

$$+c_v\dot{t}+t_0\beta_1\dot{e}+\sum_{k=1}^{m}[t_0 g_0(L_0-K_G L_1)\dot{e}^*+R_0(C_0-K_\rho C_1)\dot{t}^*]\,\delta(x_3-z_k),$$

$$\Delta u_i+\frac{e_{,i}}{1-2\nu_1}-L_1\theta_{,i}=\frac{\ddot{u}_i}{c_1^2}-\tag{12}$$

$$-\sum_{k=1}^{m}[(B_{1i,1}^*+B_{i2,2}^*-A_\rho^{(R_0)}\ddot{u}_i^*)\,\delta(x_3-z_k)+B_{i3}^*\,\delta'(x_3-z_k)]\,,i=1,\ 2,\ 3,$$

where

$$L_n=2\frac{1+\nu_n}{1-2\nu_n}(\alpha_t)_n\ \ (n=0;\ 1);\quad K_\zeta=\frac{\zeta_1}{\zeta_0};$$

$$M_n=\frac{\Omega_n}{(\lambda_t)_n};\quad \delta(\zeta)=\frac{dS(\zeta)}{d\zeta};\quad e^*=\frac{1}{2}[e(z_k+0)+e(z_k-0)];$$

$$A_\zeta^{(\xi)}=2\frac{\xi}{G_1}(1-K_\zeta),\quad B_{lj}=A_G^{(g_0)}e_{lj};$$

$$B_{jj}=A_G^{(g_0)}e_{jj}+\frac{g_0}{G_1}\left[2\left(\frac{\nu_0}{1-2\nu_0}-K_G\frac{\nu_1}{1-2\nu_1}\right)e-\right.$$

$$\left.-(L_0-K_G L_1)\,\theta\right];\quad c_1=\sqrt{\frac{G_1}{\rho_1}};$$

p_0, p_1 are characteristics of the inclusions and of the host material, z_k is the coordinate of the middle plane of the k-th layer, and m is the quantity of foreign layers.

If the solutions of (12) are found for given boundary conditions, then we determine the temperature stresses from known formulas [4] in which the physicomechanical characteristics are represented in the form

$$p(z)=p_1+(p_0-p_1)\sum_{k=1}^{m}[S_-(x_3-z_k+d)-S_+(x_3-z_k-d)].$$

Limits of applicability of the model under consideration are established in a specific example in [32]. The temperature field in a piecewise-homogeneous half-plane is studied in [33] for a partial nonideal contact (sealed crack, for instance).

Since curves of the temperature dependence of the body characteristics can be approximated by using asymmetric unit temperature functions [34] this permits application of the apparatus of generalized function to the solution of thermoelasticity problems of thermally sensitive bodies. Such an approach affords the possibility of expressing the temperature in terms of the Kirchhoff variable in nonlinear heat-conduction problems [35]. By such an approximation of the temperature dependence of the shear modulus and the substitution of the expression obtained in the equilibrium equation in displacements, the quasistatic thermoelasticity problem is reduced to the solution of a differential equation with singular coefficients of a complex argument [34, 36]. The effectiveness is shown of applying this method to studying temperature stresses in thermally sensitive bodies. A method is proposed in [37] for determining the nonstationary temperature fields in piecewise-inhomogeneous thermally sensitive bodies. The nonlinear equation of generalized heat conduction of an anisotropic body is obtained in [38]. The thermophysical characteristics for crystalline bodies are proportional to the cube of the absolute temperature at temperatures below the Debye temperature, say. In this case the nonlinear heat-conduction problem for piecewise-homogeneous bodies heated by radiation is linearized completely by using the Kirchhoff variable [38, 39], and the generalized functions apparatus is applicable for its solution. Moreover, (2) is here written differently.

The generalized thermoelasticity problem for bodies of a three-dimensional piecewise-homogeneous structure is examined in [40].

Nondestructive control methods of determining the thermophysical characteristics of materials, based on the solutions of two-dimensional nonstationary heat-conduction problems for bodies with discontinuous boundary conditions of the second kind, have been developed by the school of A. G. Shashkov [41, 42]. Consequently, solutions of two-dimensional heat-conduction problems for homogeneous, piecewise-homogeneous, isotropic, and anisotropic bodies under different boundary conditions [1, 39, 9, 10, 43] are of practical value.

NOTATION

$\lambda_{ij}{}^t(M)$, $\lambda_t(M)$, heat-conduction coefficients of an inhomogeneous anisotropic and isotropic body; $\alpha_{ij}{}^t(M)$, $\alpha_t(M)$, their temperature coefficients of linear expansion; $c_V(M)$, bulk specific heat; w_t, heat source density; e_{ij}, strain tensor components; $\rho(M)$, density; $c_{ijk\ell}$, components of the elastic stiffness tensor of an anisotropic body; $\lambda(M)$, $\mu(M)$, Lamé characteristics; e, volume expansion; $S_\pm(\zeta) = \begin{cases} 1, & \zeta > 0, \\ 0{,}5 \mp 0{,}5, & \zeta = 0 \\ 0, & \zeta < 0 \end{cases}$, asymmetric unit functions; $S(\zeta) = \begin{cases} 1, & \zeta > 0, \\ 0{,}5, & \zeta = 0 \\ 0, & \zeta < 0 \end{cases}$, symmetric unit functions; E(M), elastic modulus; $\nu(M)$, Poisson ratio; α_s, coefficient of heat elimination from the body boundary surface S; t_c^s, temperature of the external medium surrounding this surface; n_i, vector components of the external normal $\bar{n}$ to the surface S; $|\bar{n}| = 1$; Δ, Laplace operator.

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PROCESSES OF HEAT, MASS, AND MOMENTUM TRANSFER IN DISPERSE MEDIA

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The first new-concept technological plants to utilize disperse media for intensification of the processes of annealing of zinc concentrates, drying of fine-grained materials, and the combustion of solid fuel in fluidized beds and vibrating fluidized beds were developed on the basis of experimental studies without any incisive theoretical research efforts. This meant that theoretical investigations in the given field were dictated by practice.

However, such a rift between theory and practice quickly led to miscalculations and errors in the development of certain technologies (rapid heating and combustion of fine-grained fuel), which could only have adverse implications for the overall development of the problem.

With these considerations in mind, a research program has been undertaken recently at the Department of Theoretical Heat Engineering of the Ural Polytechnic Institute with a view toward the development of theoretical methods for analyzing the behavior of the indicated

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